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ARTICLE

The width of the flux tube in SU(3) pure gauge theory

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Abstract: In this paper, we present results obtained from the flux tubes between a quark and an antiquark in SU(3) pure gauge lattice field theory. We fit the transverse distribution of the energy density and the parallel components of the chromoelectric and chromomagnetic fields with several functions and used the coefficients obtained from the fit for computation of the width of the flux tube. Subsequently, we looked into in the dependence of the width of the flux tube on the distance between the static quark-antiquark sources and temperatures. Finally, we compared our results with those from other similar studies. We observed that the numerical value of the width of the flux tube computed from the pure gauge theory is larger than that from the full QCD theory, and the width of the flux tube widened until T_c, and then decreased above the critical temperature.

Keywords: *Lattice QCD; SU(3) pure gauge theory; width of the flux tube; simultaneous fit;*

INTRODUCTION

Quantum chromodynamics (QCD) is the fundamental quantum field theory of the interaction between quarks mediated by gluons, which is based on color symmetry [1].

The quarks are fundamental particles that make up composite hadrons, such as proton and neutron that contain elementary particles called u and d quarks.

These quarks are fundamental particles of a matter. Quarks are combined mediately by gluons that carry strong interaction within a hadron. They never exist in isolation due to the quark confinement phenomenon in which the potential energy between the quark and antiquark increases linearly with the interquark distance. Therefore, finding answers to the unexplained problems in the interaction of two quarks is more complicated. Some understanding of confinement and deconfinement in QCD remain as the keys problems in particle physics. Quark confinement arises from the nature of the strong force. A critical concept that lattice physicists are currently trying to explain is the quark confinement phenomenon.

Gluons create a flux tube as they carry interaction. The flux tube system can be described by the correlation function of the plaquette and Polyakov loop [1-7].

$$f_{\mu\nu}(R,x) = \frac{\beta}{a^4}\left[\frac{\langle L(0)L^{+}(R)\square_{\mu\nu}(x)\rangle}{L(0)L^{+}(R)} - \langle\square_{\mu\nu}\rangle(x)\right] \qquad (1)$$

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Here $L(0)$ is a Polykov loop, and its Hermitian conjugate is $L^{+}(R)$, and $\square_{\mu,\nu}$ represents plaquette, β is coupling constant, R is distance between the two quarks, a is lattice spacing.

The study of gauge theories on space-time that has been discretized into a lattice is described by lattice gauge theory. Pure gauge and full QCD theories [7] are used to study the behavior of strong interaction in the field of particle physics. Pure gauge theory focuses solely on the interactions and behavior of these gluons without considering the effects of quarks. Full QCD is the complete theory that incorporates both gluons and quarks.

In this study, we used SU(3) pure gauge theory, that describes the interactions between the gluons.The advantage of the pure gauge theory is that the study of removing quarks provides a simplified approach. By eliminating the complexity introduced by quarks, calculations and theoretical analysis can be more tractable. Numerical studies with SU(3) Yang-Mills theories have established a flux tube formation between a quark and an antiquark.

Our paper is divided into five sections: introduction, the width of the flux tube, simulation details, results and conclusions. Section four presents the method used in calculating the width of the flux tube. Section Five presents the results of our study, including the comparison of the pure gauge and full QCD theories, and the dependence of the width of the flux tube on the distance between the quarks and temperature. The last section summarizes our findings and provides some concluding remarks.

The width of the flux tube

We measured the parallel and perpendicular components of chromoelectric and chromomagnetic fields surrounding the quarks. These components are used to calculate the total chromomagnetic and chromoelectric fields inside the flux tube. Also, the energy density is calculated from those fields. Energy density is a very important criterion, because, the transverse distribution of the energy density defines the width of the flux tube, as shown in Figure 1.

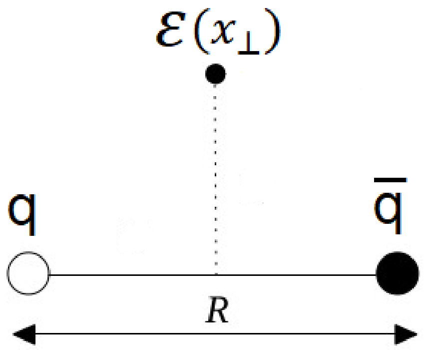

Figure 1. The energy density at transverse distance $x_\perp$ from the middle of the axis connecting the quarks

There are two ways to compute the width of the flux tube. First, it can be calculated by the following equation (2). Here, we can compute the width of the flux tube using the transverse distribution of the energy density with the following formula:

$$D = \frac{\int dx_\perp^2 x_\perp x_\perp^2 \varepsilon(x_\perp)}{\int d^2 x_\perp \varepsilon(x_\perp)} \qquad (2)$$

Secondly, it is defined as

$$D = \frac{\int dx_\perp^2 x_\perp x_\perp^2 (E_\parallel^2 - B_\parallel^2)}{\int d^2 x_\perp (E_\parallel^2 - B_\parallel^2)} \qquad (3)$$

where $B_\parallel$ and $E_\parallel$ represent the parallel components of chromoelectric and chromomagnetic fields.

We will discuss in more detail about fit functions that are fitted with the data of energy density and parallel components of color fields, and computation of the width of the flux tube using coefficients obtained from the fit in section IV.

Flux tube measurement

We measured the components of the chromomagnetic and the chromoelectric fields in the flux tube between the quark and antiquark using SU(3) pure gauge theory on the lattice. Here, the volume of the lattice is $32^3 \times 8$. Measurements were performed at several temperatures, and the statistical noise was reduced by the Gradient Flow method at several values of flow time. At each temperature, 1000 configurations were created for the pure gauge theory. The measurement parameters are shown in Table 1.

Table 1. Measurement parameters

$N_s \times N_\tau$	T/T_c	N_{conf}	R
$32^3 \times 8$	0.859	1000	4a-16a
	0.970		
	1.030		
	1.090		
	1.233		
	1.867		

We used the ParallelGPUCode package program in our computation.

Calculation of the width of the flux tube

We have used the two cases which are mentioned in the second section to calculate the width of the flux tube. Let's consider the first case, which used the transverse distribution of the energy density. Here we applied two exponential functions $f_1(x)$ and $f_2(x)$, and a Coulomb-like function $f_3(x)$ to fit with the data of the transverse distribution of the energy density. Exponential functions [8] are given by

$$f_1(x) = ae^{-bx} + c, \tag{4}$$

from here, the width of the flux tube is defined as

$$D^2a^{-2} = \frac{6}{b^2}, \tag{5}$$

and

$$f_2(x) = ae^{-bx^2} + c, \tag{6}$$

from here, the width of the flux tube is defined as

$$D^2a^{-2} = \frac{1}{b^2}, \tag{7}$$

and a Coulomb-like function [8] is given by

$$f_3(x) = \frac{a}{(b+x_\perp^2)^3}, \tag{8}$$

the width of the flux tube is defined as

$$D^2a^{-2} = b\,, \tag{9}$$

In another case, we used the simultaneous fit that fits the two different data with the two different functions simultaneously. Here, we chose the function [3]

$$\frac{1}{2\beta}E_\parallel^2(R,x_\perp) = \frac{a_1}{(a_2+x_\perp^2)^3} + \frac{a_3}{(a_4+x_\perp^2)^3} \tag{10}$$

to fit with the parallel component of the chromoelectric field and the function

$$\frac{-1}{2\beta}B_\parallel^2(R,x_\perp) = \frac{a_1}{(a_2+x_\perp^2)^3} \tag{11}$$

to fit with parallel component of the chromomagnetic field. Then, the width of the flux tube was calculated as follow

$$D^2a^{-2} = a_4\,. \tag{12}$$

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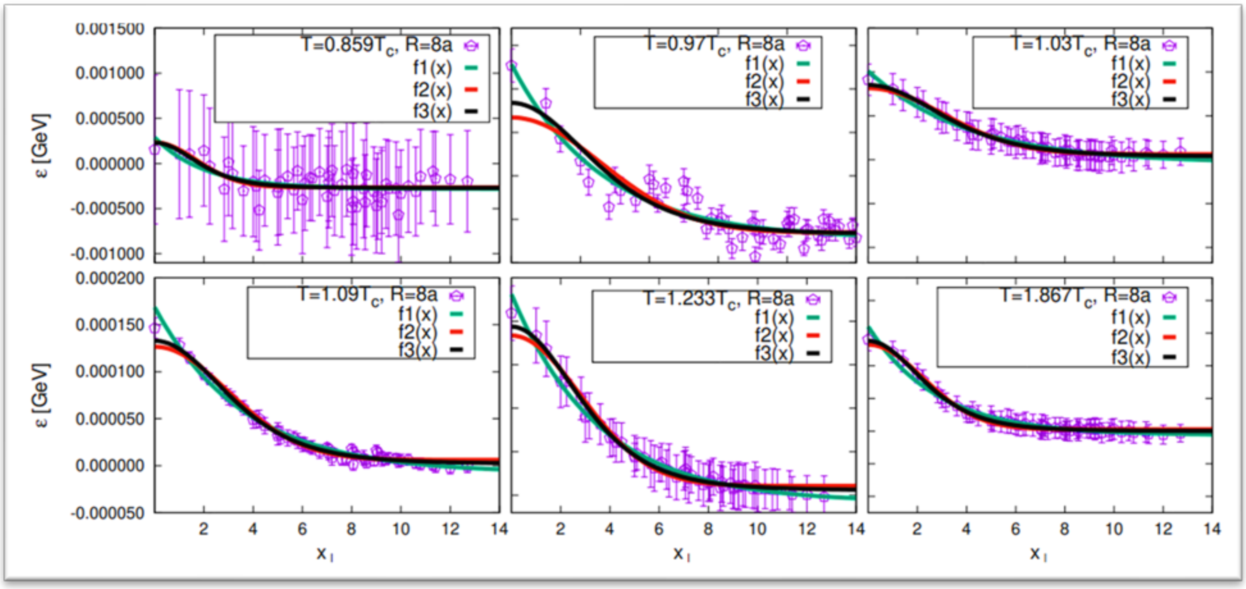

Figure 2. Comparison of $f_1(x), f_2(x)$ and $f_3(x)$ fit functions

Moreover, we compared them to choose a suitable function. The results that three functions were fitted with the data of transverse distribution of the energy density as shown in Figure 2. Here the distance between the quarks is R=8a for all temperatures.

From the above comparison, one can see that these functions were in general agreement, and we chose the one function in which the number of degrees of freedom χ^2 equals approximately 1. The fit parameters are shown in Table 2.

Table 2. Numerical values of the fit parameters with the eq.3

$f_3(x)$					
β	T/T_c	R	χ	b	δb
5.970	0.859	4a	0.227	5.946	2.438
		6a	0.321	1.527	5.644
		8a	0.328	6.682	1.047
		10a	0.317	1.725	2.384
		12a	0.386	1.939	3.475
		14a	0.288	10.533	26.260
		16a	0.409	2.650	4.068
6.043	0.970	4a	2.288	6.304	0.398
		6a	2.448	14.143	1.540
		8a	1.420	27.341	5.501
		10a	3.093	57.783	22.80
		12a	4.414	129.836	140.70
		14a	2.598	257.648	369.10
		16a	0.173	19.142	10.720
6.081	1.030	4a	1.018	6.304	0.398
		6a	2.448	14.143	1.540
		8a	1.420	27.341	5.501
		10a	3.093	57.783	22.80
		12a	4.414	129.836	140.70
		14a	2.598	257.648	369.10
		16a	0.173	23.866	14.360

6.117	1.090	4a	1.867	6.983	0.331
		6a	1.335	17.414	1.091
		8a	0.623	42.547	2.387
		10a	0.585	37.454	2.771
		12a	0.480	46.772	7.059
		14a	0.480	76.539	27.680
		16a	0.642	221.697	156.23
6.200	1.233	4a	1.749	8.155	0.365
		6a	0.462	17.248	0.762
		8a	0.212	37.187	1.614
		10a	0.235	56.643	4.046
		12a	0.162	59.785	8.411
		14a	0.188	614.683	352.3
		16a	0.475	0.556	10.66
6.500	1.856	4a	1.841	8.283	0.355
		6a	0.415	14.419	0.510
		8a	0.183	23.768	0.776
		10a	0.107	32.302	1.774
		12a	0.173	69.747	12.90
		14a	0.242	635.237	480.20
		16a	0.329	1.557	2.310

Now, we introduce the results of the simultaneous fit with data of the parallel components of the chromo fields. Data and fitting results for the parallel components of chromoelectric and chromomagnetic fields are plotted in Figure 3.

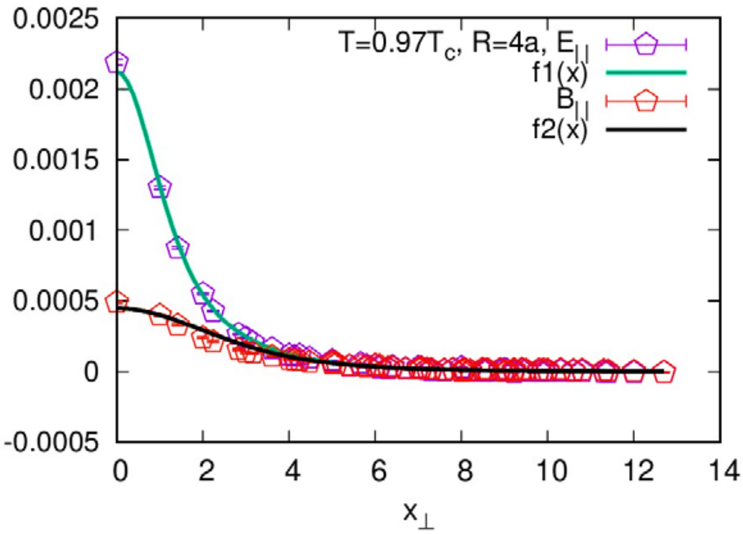

Figure 3. Simultaneous fit with the parallel components of chromoelectric and chromomagnetic fields

We calculated the width of the flux tube using the coefficients obtained from the fit that fits the transverse distribution of energy density with the curve of $f_3(x)$ fit function and the data of parallel components of chromo fields with the simultaneous fit function. Then we compared the results of the widths of the flux tube from these two fit functions to determine the best fit. The compared results are shown in Figure 4.

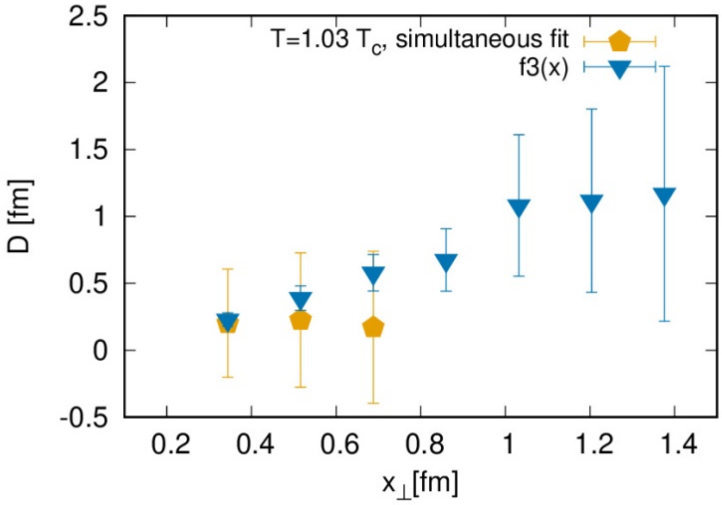

Figure 4. Comparison of widths of the flux tube from simultaneous fit and $f_3(x)$ at various distances

The simultaneous fit was a perfect match with the data for short distance. But we were unable to calculate the width of the flux tube for the larger distances because the simultaneous fit did not work for distances between the quarks larger than 0.75 fm. So, we further used the results computed from the energy density

RESULTS

We plotted the temperature dependence of the width of the flux tube for $R = 0.6$ fm as in Figure 5.

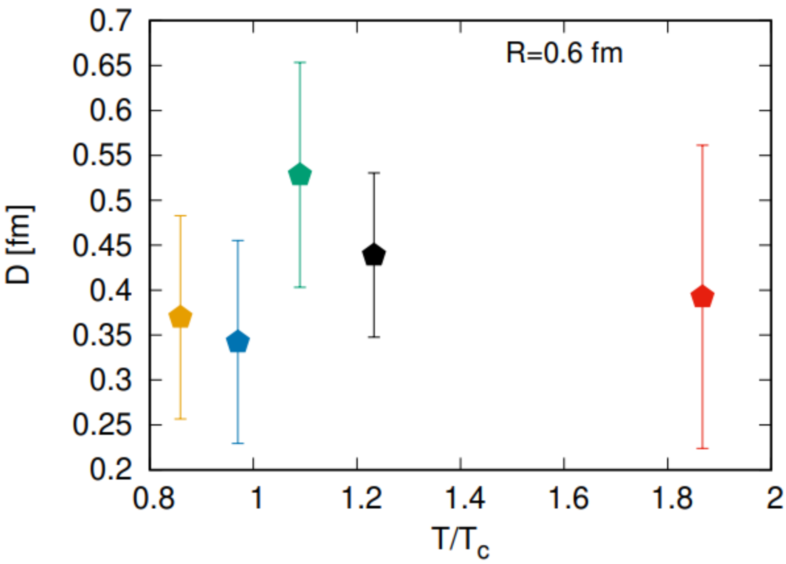

Figure 5. The width of the flux tube as a function of temperature

From the plot, we can demonstrate that the width of the flux tube increased until T_c, then decreased above the critical temperature. In Figure 6, we compared the results of the width of the flux tube as a function of the distance between a quark and an antiquark with the results from pure gauge theory in Ref.[2] to double check if our results are accurate. The numerical values were similar to the results from that work.

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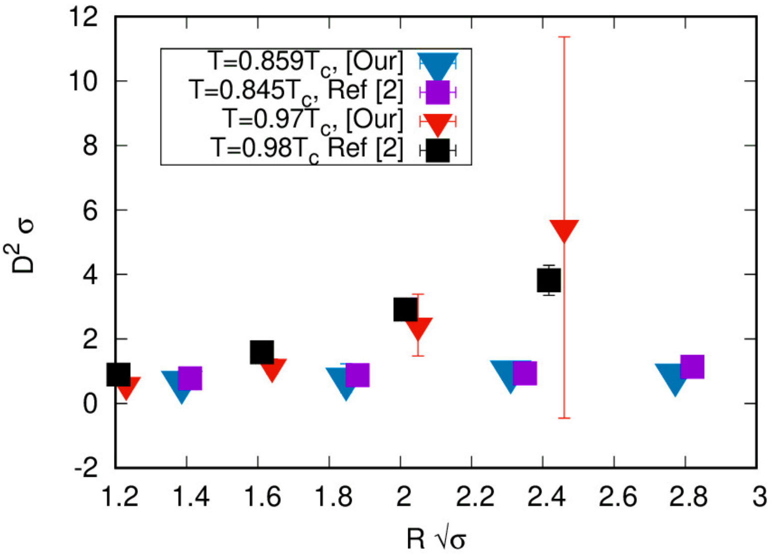

Figure 6. A comparison of results for pure gauge theories. The width of the flux tube as a function of distance between the quarks

Finally, we compared the width of the flux tube in the SU(3) pure gauge theory and the results obtained from full QCD [5] to see if there were any disparities between these two theories. The comparison of this analysis are illustrated in Figures 7a and 7b.

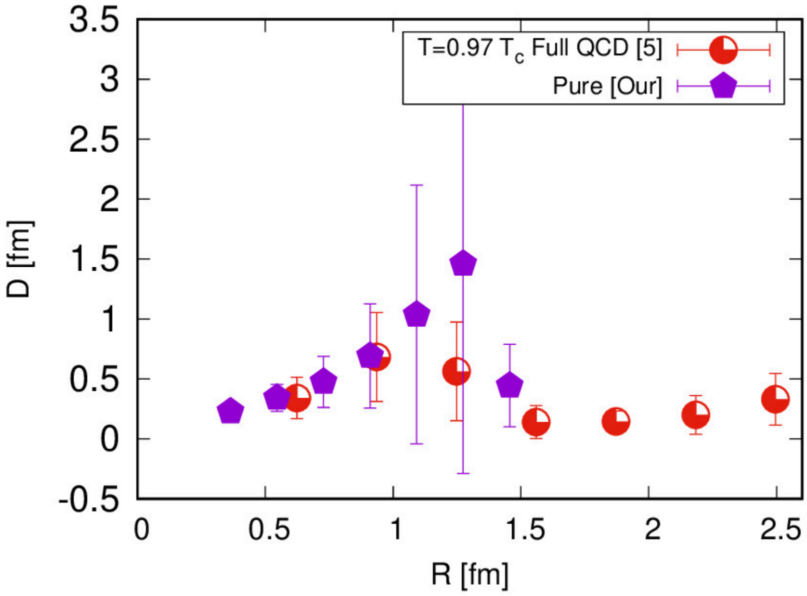

Figure 7a. Comparison of widths from pure gauge and full QCD theories. The width of the flux tube as a function of the distance between the quarks in deconfined phase

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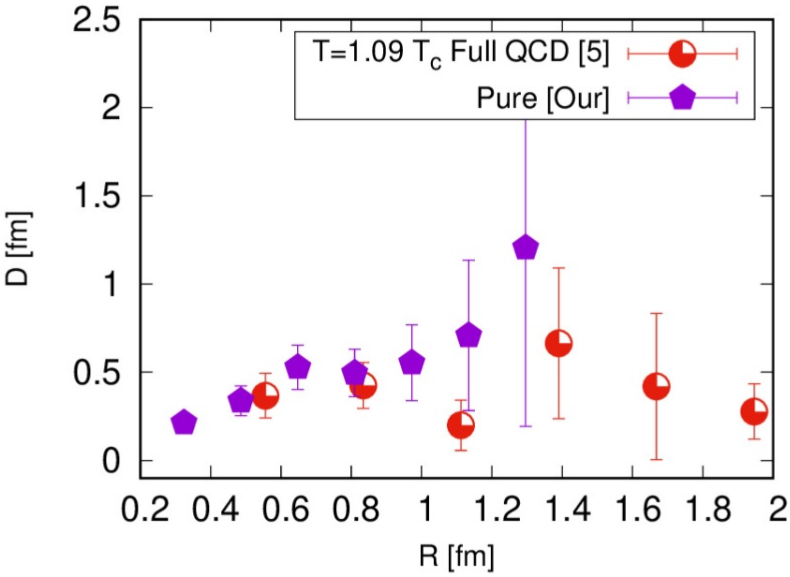

Figure 7b. Comparison of widths from pure gauge and full QCD theories. The width of the flux tube as a function of the distance between the quarks in confined phase

In the case of pure gauge theory, it was observed that the width of the flux tube increases depending on the distance between the quarks. However, in full QCD theory, there exists an assumption that the width of the flux tube widens due to the dynamical quarks until the separation reaches approximately 1 fm. As the separation increases in the range between 1 fm and 1.5 fm, hadronization takes place with the presence of dynamical fermions. And for the quark separation larger than 1.5 fm the width curve becomes flat within the error bars [5].

Finally, we compared our results of the width of the flux tube as a function of temperature at the fixed value of R=0.6 fm, with the results obtained from the full QCD theory.

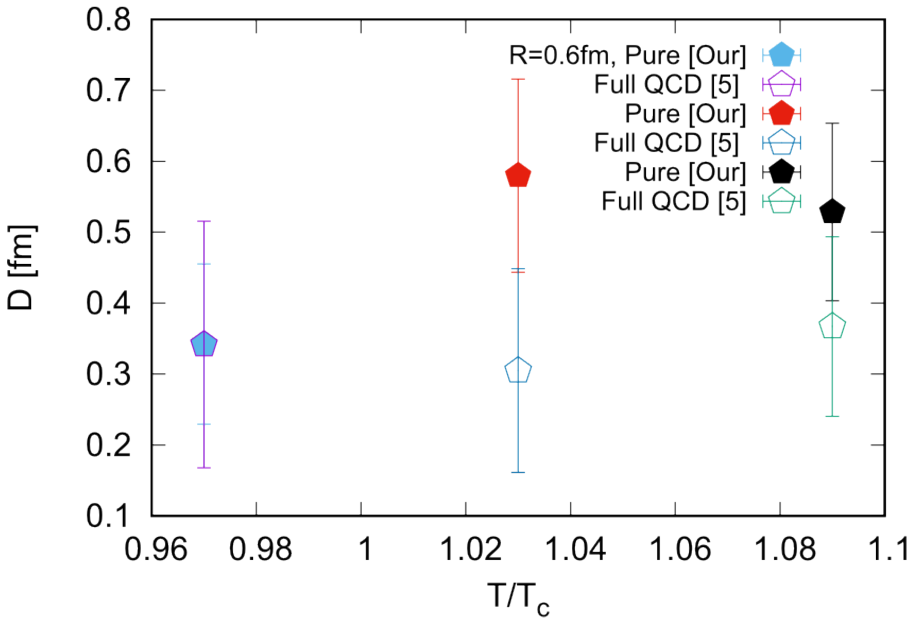

Figure 8. Comparison of widths from pure gauge and full QCD theories. The width of the flux tube as a function of the temperature

As can be seen from Figure 8 the width of the flux tube is similar for both full QCD and the pure gauge theory at $0.97T_c$. This implies that there is no significant difference in the width of the flux tube between the two theories at that temperature. However, at a higher temperature of $1.03T_c$, there is a difference in the width of the flux tube between the two theories. In full QCD, the value of the width of the flux tube is 0.3, while in the pure gauge theory, it measured about 0.6. Furthermore, at $1.09T_c$, the width of the flux tube remains different between the two theories. In full QCD, the value is 0.32, while in the pure gauge theory, it measured 0.5. This implies that the presence of quarks in full QCD affects the width of the flux tube, resulting in a narrower tube compared to the pure gauge theory at this temperature.

CONCLUSIONS

We measured the components of the chromomagnetic and chromoelectric fields produced by quark and antiquark sources at several temperatures above and below the critical temperature. We calculated the width of the flux tube using the values of the free parameters obtained from the fit. We studied the dependence of the width of the flux tube on the interquark distance and temperature.

For the dependence of the temperature: The width of the flux tube increases until T_c, then it decreased as the temperature increased above $1T_c$.

For the dependence of the distance between the quarks: The width of the flux tube increased as the distance between the quarks increased.

Our results indicate that the flux tube widens until the critical temperature, but above the critical temperature, it narrows as the temperature increases.

Also, we compared our results with the results from the other studies using pure gauge [2] and full QCD [5] theories. From the comparison we found that results of pure gauge and full QCD theories are distinguished. In other words, when comparing the two theories, the width of the flux tube, calculated using thes pure gauge theory, was found to have greater value than the width calculated using the full QCD theory above the critical temperature. This suggests that there may be differences in the properties of the flux tube between the pure gauge theory and the full QCD theory. Also, dependence on the interquark distance was different

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