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# A study of Indium phosphide and line graph of subdivision graph of *H*-naphtalenic nano-sheet via irregularity indices

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## ABSTRACT

In this article, we have taken the molecular graph of indium phosphide *InP[m,n]* and line graph of subdivision graph of *H*-naphtalenicnano-sheet *L(S(H(r,s)))*. Irregularity indices play an important role to describe the quantitative characterization of the non-regular graphs. In various problems and applications, particularly in subject of chemistry and material engineering irregular indices have so many uses, thus it is very important to know about the irregularity of a molecular structure. Moreover, the evaluation of the irregularity of graphs is an important not only for QSPR and QSAR but also very effective for the measuring the entropy, melting and boiling points, enthalpy of vaporization and toxicity. We have also discussed the graphical behaviors of the above indicated structures.

**Keywords**: Topological indices, irregular indices, indium phosphide, *H*-naphtalenic, molecular structure

## INTRODUCTION

Corona virus 2 (SARS-CoV-2) is a contagious and caused form submicroscopic infectious agent virus [1-3]. Basically, nano-sheets are found in 2D nano-structures having thickness from 1 to 100 nm [1-4]. Nano-sheets having wonderful, physical, biological, electronic and chemical characteristics that are very effective for catalysis, storage, nano-bio-technology, electronics, optical and dielectric-associated problems. Thus, it is very important to describe these nano-structures to get understanding in the topology and increase their physical characteristics. Modern technologies depend upon the nano-materials/naphtalenic nano-sheet are very good in controlling the problems in usual drug transportation namely: weakly solubility, poison and weakly deliver design of drugs. The physical and chemical characteristics of these nano-sheets rely on their structure. *H*-Naphtalenic nano-sheet is denoted by (*H(r,s)*)and it contains alternating hexagons $C_6$, $C_4$ and octagons$C_8$. The number of vertices in (*H(r,s)*) is 10 rs, where r denotes the number of paired hexagons in every alternate row with $C_4$ cycle, and s denotes the number of rows consisting $C_4$. TIs of *H*-Naphtalenic nano-sheet are studied in [5]. In the present calculations, we employed edge partition and vertex partition tools of graph theory [6]. The graph of *H*-Naphtalenic nano-sheet is denoted by *H*(*r,s*), and line graph of subdivision graph is denoted by *L(S(H(r, s)))*. Indium phosphide, is denoted by InP lie in group IIIA-VA, a semiconductor and its decomposing into bomb calorimeter give us two types of phosphorous i.e. red and black [7]. Crystallizes form of Inp is cubic form which is shown in Fig.1. InP is used commercially from so many years due to having band gap 1.35 Ev, complete composition, low toxic nature, shape and solution development potential. Inp nano-crystal has no cadmium and less-toxic nano-crystal with excellent characteristics like top thermal conductivity, stokes shift emission [8], near IR emitter, and best PL quantum turnout [9]. Inp is also used in the photo electro chemistry because it has band gap 1.35 Ev which is capture from the solar radiation by lowering the reflection of photons, resulting in separating of water in solar. Moreover, by using InP nano-wire arrays on Si give excellent behavior in this field because of less cost, top surface area, and catalytic characteristics [10]. Nano-structured of InP indicate its incoming applications in the field of nano-photonic and optoelectronic devices as its excellent performance in harvesting and energy saving characteristics [11]. An extensible fabricated synapse based on InP was shown for upcoming applications with neuromorphic developments [8].

In chemical graphs vertices are represented by atoms and edges are represented by chemical bonds [12, 13]. The important famous and very commonly used graph is the line graph. If *H* is a non-empty graph, then we represent line graph by *L*(*H*) and it has set of edges that lies in *H* as its vertex set. In *L*(*H*) two vertices are said to be adjacent whenever corresponding edges of graph G are adjacent. In our study, we have denoted the degree of *g* and h by $d_g$ and $d_h$, respectively, and *gh* ∈ *E*(*G*). In this paper, many irregular indices of the molecular graph of the chemical compound of *InP*[*m,n*] and L(S(H(r, s))) are determined.

***Equations work***: In this section, used some equations to determine the irregular indices for the molecular graph of indium phosphide *InP[m,n]* and line graph of subdivision graph of *H*-naphtalenic nano-sheet *L*(*S*(*H*(*r*,*s*))). The irregular indices based on vertex degree are given below from Eq. 1.1 to Eq 1.16 respectively [14, 15-24].

$$VAR(G) = \frac{M_1(G)}{k} - \left(\frac{2l}{k}\right)^2 \quad (1.1)$$

$$AL(G) = \sum_{gh\in E(G)} |d_g - d_h| \quad (1.2)$$

$$IR1(G) = F(G) - \frac{2l}{k} M_1(G) \quad (1.3)$$

$$IR2(G) = \sqrt{\frac{M_2(G)}{l}} - \frac{2l}{k} \quad (1.4)$$

$$IRF(G) = F(G) - 2M_2(G) \quad (1.5)$$

$$IRFW(G) = \frac{IRF(G)}{M_2(G)} \quad (1.6)$$

$$IRA(G) = \sum_{gh\in E(G)} \left(d_g^{-\frac{1}{2}} - d_h^{-\frac{1}{2}}\right)^2 \quad (1.7)$$

$$IRA(G) = \sum_{gh\in E(G)} \left(d_g^{1/2} - d_h^{1/2}\right)^2 \quad (1.8)$$

$$IRA(G) = \sum_{gh\in E(G)} \frac{\sqrt{d_g d_h}}{l} - \frac{2l}{k} \quad (1.9)$$

$$IRDIF(G) = \sum_{gh\in E(G)} \left|\frac{d_g}{d_h} - \frac{d_h}{d_g}\right| \quad (1.10)$$

$$IRL(G) = \sum_{gh\in E(G)} |lnd_g - lnd_h| \quad (1.11)$$

$$IRLU(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{min(d_g, d_g)} \quad (1.12)$$

$$IRLF(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{\sqrt{d_g d_h}} \quad (1.13)$$

$$IRLA(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{d_g + d_h} \quad (1.14)$$

$$IRDI(G) = \sum_{gh\in E(G)} ln\{1 + |d_g - d_h|\} \quad (1.15)$$

$$IRGA(G) = \sum_{gh\in E(G)} \frac{ln(d_g + d_h)}{2\sqrt{d_g d_h}} \quad (1.16)$$

Moreover, first and second Zagreb indices are defined as [25]

$$M_1(G) = \sum_{gh\in E(G)} (d_g + d_h),$$

$$M_2(G) = \sum_{gh\in E(G)} (d_g \times d_h).$$

Forgotten index, is defined as [26]

$$F(G) = \sum_{gh\in E(G)} ({d_g}^2 + {d_h}^2).$$

## RESULTS AND DISCUSSIONS

***Results on indium phosphide InP[m,n]***: In this section, we compute some results on irregular indices for indium phosphide *InP[m,n]* which will be very effective to analyze the graphical behaviors of *InP[m,n]*.

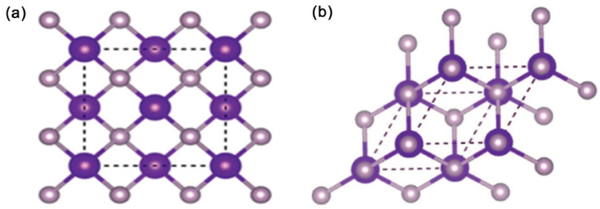


Fig. 1. Structural cell of indium phosphide *InP[m,n]:* **(a)** front view and **(b)** general view

Table1. Edge partition of *InP[m,n]*

| Types of edges ($d_g$, $d_h$) | Count of edges |
|---|---|
| (1,4) | 4*m* + 4*n* - 4 |
| (2,4) | 4*mn*+4*m*+4*n* |
| (4,4) | 8*mn*-4*m*-4*n* |

***Theorem***

Let G denote *InP[m,n]*, where *InP[m,n]* represent indium phosphide, then we compute the followings indices:

(1) $VAR(G) = \dfrac{36m^2 - 36n^2 + 384m^2n + 144n^2m - 36m - 84n + 880m^2n^2 - 24mn - 40}{(10mn + 3m + 3n + 2)^2}$

(2) $AL(G) = -20m - 20n - 8mn + 12$

(3) $IR1(G) = \dfrac{60m^2 + 60n^2 + 824m^2n + 1592n^2m - 164m - 164n + 544m^2n^2 + 752mn - 136}{(10mn + 3m + 3n + 2)}$

(4) $IR2(G) = \sqrt{\dfrac{-m - n + 10mn - 1}{mn}} - \dfrac{32mn}{10mn + 3m + 3n + 2}$

(5) $IRF(G) = 52m + 52n + 16mn - 36.$

***Proof:***

(1) Using Eq. 1.1 and Table 1, we get

$$VAR(G) = \frac{M_1(G)}{k} - \left(\frac{2l}{k}\right)^2$$

$$= \frac{12m - 12n + 88mn - 20}{(10mn + 3m + 3n + 2)} - \left(\frac{32mn}{10mn + 3m + 3n + 2}\right)^2$$

$$= \frac{(12m - 12n + 88mn - 20)(3m + 3n + 10mn + 2) - 32mn}{(10mn + 3m + 3n + 2)^2}$$

$$= \frac{36m^2 - 36n^2 - 384m^2n - 144mn^2 - 36m - 84n + 880m^2n^2 - 24mn}{(10mn + 3m + 3n + 2)^2}$$

(2) Using Eq. 1.2 and Table 1, we get

$$AL(G) = \sum_{gh\in E(G)} |d_g - d_h|$$

$$= |1 - 4|(4m + 4n - 4) + |2 - 4|(4mn + 4m + 4n) + |4 - 4|(8mn - 4m$$

$$= -20m - 20n - 8mn + 12.$$

(3) Using Eq. 1.3 and Table 1, we have

$$IR1(G) = F(G) - \frac{2l}{k} M_1(G)$$

$$= (20m + 20n + 336mn - 68) - \frac{2(16mn)}{(10mn + 3m + 3n + 2)} (12m - 12n + 88mn - 20)$$

$$= \frac{60m^2 + 60n^2 + 824m^2n + 1592n^2m - 164m - 164n + 544m^2n^2 + 752mn - 136}{(10mn + 3m + 3n + 2)}$$

(4) Using Eq. 1.4 and Table 1, we conclude

$$IR2(G) = \sqrt{\frac{M_2(G)}{l}} - \frac{2l}{k}$$

$$= \sqrt{\frac{-16mn - 16n + 160mn - 16}{16mn}} - \frac{32mn}{(10mn + 3m + 3n + 2)}$$

$$= \sqrt{\frac{-mn - n + 10mn - 1}{mn}} - \frac{32mn}{(10mn + 3m + 3n + 2)}.$$

(5) Using Eq. 1.5 and Table 1, we get

$$IRF(G) = F(G) - 2M_2(G)$$

$$= (20m + 20n + 336mn - 68) - 2(-16mn - 16n + 160mn - 16)$$

$$= 52m + 52n + 16mn - 36.$$

***Theorem***

Suppose *G* denote *InP*[m,n], where *InP*[m,n] represent indium phosphide, then we compute the followings indices:

(1) $IRFW(G) = \frac{13m + 13n + 4mn - 9}{-4m - 4n + 40mn - 4}$

(2) $IRA(G) = \left(4 - \frac{4}{\sqrt{2}}\right)m + \left(4 - \frac{4}{\sqrt{2}}\right)n + \left(-3 - \frac{4}{\sqrt{2}}\right)mn + 1$

(3) $IRB(G) = (28 - 16\sqrt{2})m + (24 - 16\sqrt{2})mn + (28 - 16\sqrt{2})n - 4$

(4) $IRC(G) = \frac{(-24 + 4\sqrt{8})m + (32 + 4\sqrt{8})mn + (-24 + 4\sqrt{8})n - 8}{16mn} - \frac{32mn}{10mn + 3m + 3n + 2}$

(5) $IRDIF(G) = -27m - 27n - 12mn + 15.$

***Proof:***

(1) Using Eq. 1.6 and Table 1, we have

$$IRFW(G) = \frac{IRF(G)}{M_2(G)}$$

$$= \frac{52m + 52n + 16mn - 36}{-16mn - 16n + 160mn - 16}$$

$$= \frac{13m + 13n + 4mn - 9}{-4m - 4n + 40mn - 4}$$

(2) Using Eq. 1.7 and Table 1, we have

$$IRA(G) = \sum_{gh\in E(G)} \left(d_g^{-1/2} - d_h^{-1/2}\right)^2$$

$$= \left(1 - \frac{1}{\sqrt{4}}\right)^2 (4m + 4n - 4) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{4}}\right)^2 (4mn + 4m + 4n)$$

$$+ \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{4}}\right)^2 (8mn - 4m - 4n)$$

$$= \left(4 - \frac{4}{\sqrt{2}}\right)m + \left(4 - \frac{4}{\sqrt{2}}\right)n + \left(-3 - \frac{4}{\sqrt{2}}\right)mn + 1$$

(3) Using Eq. 1.8 and Table 1, we get

$$IRB(G) = \sum_{gh\in E(G)} \left(d_g^{1/2} - d_h^{1/2}\right)^2$$

$$= \left(\sqrt{1} - \sqrt{4}\right)^2 (4m + 4n - 4)$$
$$+ \left(\sqrt{2} - \sqrt{4}\right)^2 (4mn + 4m + 4n)$$
$$+ \left(\sqrt{4} - \sqrt{4}\right)^2 (8mn - 4m - 4n)$$

$$= (28 - 16\sqrt{2})m + (24 - 16\sqrt{2})mn + (28 - 16\sqrt{2})n - 4$$

(4) Using Eq. 1.9 and Table 1, we conclude

$$IRC(G) = \sum_{gh\in E(G)} \frac{\sqrt{d_g d_h}}{l} - \frac{2l}{k}$$

$$= \frac{(\sqrt{1\times 4})(4m + 4n - 4) + (\sqrt{2\times 4})(4mn + 4m + 4n) + (\sqrt{4\times 4})(8mn - 4m - 4n)}{16mn}$$

$$- \frac{2(16mn)}{10mn + 3m + 3n + 2}$$

$$= \frac{(-24 + 4\sqrt{8})m + (32 + 4\sqrt{8})mn + (-24 + 4\sqrt{8})n - 8}{16mn}$$

$$- \frac{32mn}{10mn + 3m + 3n + 2}.$$

(5) Using Eq. 1.10 and Table 1, we have

$$IRDIF(G) = \sum_{gh\in E(G)} \left|\frac{d_g}{d_h} - \frac{d_h}{d_g}\right|$$

$$= \left|\frac{1}{4} - \frac{4}{1}\right| (4m + 4n - 4)$$
$$+ \left|\frac{2}{4} - \frac{4}{2}\right| (4mn + 4m + 4n)$$
$$+ \left|\frac{4}{4} - \frac{4}{4}\right| (8mn - 4m - 4n)$$

$$= -27m - 27n - 12mn + 15.$$

***Theorem***

Let *G* denote *InP*[*m,n*], where *InP*[*m,n*] represent indium phosphide, then we compute the followings indices:

(1) $IRL(G) = 8.3164m + 8.3164n + 2.772mn - 5.544$

(2) $IRLU(G) = 16m + 16n + 4mn - 12$

(3) $IRLF(G) = (6+\sqrt{8})m + (6+\sqrt{8})n + \sqrt{8}mn - 4$

(4) $IRLA(G) = \frac{56}{15}m + \frac{56}{15}n + \frac{4}{3}mn - 4$

(5) $IRDI(G) = 9.9384m + 9.9384n + 4.3944mn - 5.544$

(6) $IRGA(G) = 1.1275m + 1.1275n + 0.2355mn - 0.892$

***Proof:***

(1) Using Eq. 1.11 and Table 1, we have

$$IRL(G) = \sum_{gh\in E(G)} |ln d_g - ln d_h|$$

$$= |ln1 - ln4|(4m+4n-4) + |ln2 - ln4|(4mn+4m+4n) + |ln4 - ln4|(8mn-4m-4n)$$

$$= 8.3164m + 8.3164n + 2.772mn - 5.544$$

(2) Using Eq. 1.12 and Table 1, we have

$$IRLU(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{min(d_g, d_g)}$$

$$= \frac{|1-4|}{min(1,4)}(4m+4n-4) + \frac{|2-4|}{min(2,4)}(4mn+4m+4n) + \frac{|4-4|}{min(4,4)}(8mn-4m-4n)$$

$$= 16m + 16n + 4mn - 12.$$

(3) Using Eq. 1.13 and Table 1, we get

$$IRLF(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{\sqrt{d_g d_h}}$$

$$= \frac{|1-4|}{\sqrt{1\times 4}}(4m+4n-4) + \frac{|2-4|}{\sqrt{2\times 4}}(4mn+4m+4n) + \frac{|4-4|}{\sqrt{4\times 4}}(8mn-4m-4n)$$

$$= (6+\sqrt{8})m + (6+\sqrt{8})n + \sqrt{8}mn - 4.$$

(4) Using Eq. 1.14 and Table 1, we conclude

$$IRLA(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{d_g + d_h}$$

$$= \frac{|1-4|}{1+4}(4m+4n-4) + \frac{|2-4|}{2+4}(4mn+4m+4n) + \frac{|4-4|}{4+4}(8mn-4m-4n)$$

$$= \frac{56}{15}m + \frac{56}{15}n + \frac{4}{3}mn - 4.$$

(5) Using Eq. 1.15 and Table 1, we have

$$IRDI(G) = \sum_{gh\in E(G)} ln\{1 + |d_g - d_h|\}$$

$$= ln\{1 + |1-4|\}(4m+4n-4) + ln\{1+|2-4|\}(4mn+4m+4n) + ln\{1+|4-4|\}(8mn-4m-4n)$$

$$= 9.9384m + 9.9384n + 4.3944mn - 5.544.$$

(6) Using Eq. 1.16 and Table 1, we get

$$IRGA(G) = \sum_{gh\in E(G)} \frac{ln(d_g + d_h)}{2\sqrt{d_g d_h}}$$

$$IRGA(G) = \frac{ln(1+4)}{2\sqrt{1\times 4}}(4m+4n-4) + \frac{ln(2+4)}{2\sqrt{2\times 4}}(4mn+4m+4n) + \frac{ln(4+4)}{2\sqrt{4\times 4}}(8mn-4m-4n)$$

$$= 0.16328r + 0.08164s - 0.16328.$$

### *Results on L(S(H(r,s)))*

In this section, we compute some results on *L(S(H(r,s)))* by using irregular indices and the graphical behaviors of *L(S(H(r,s)))* is shown at the end.

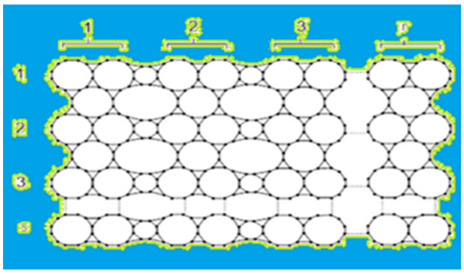


Fig. 2. Line graph of subdivision graph of *H*-naphtalenic nano-sheet *L(S(H(r, s)))*

Table 2. Edge partition of *L(S(H(r, s)))*

| Types of edges ($d_g$, $d_h$) | Count of edges |
|---|---|
| (1,4) | 4*r* + 6*s* + 4 |
| (2,4) | 8*r* + 4*s* - 8 |
| (4,4) | 45*rs* - 22*r* - 20*s* + 4 |

### *Theorem*

Let *G* denote*L(S(H(r,s)))*, where *L(S(H(r,s))* be the line graph of subdivision graph of *H*-naphtalenicnano-sheet, then we compute the followings indices:

(1) $VAR(G) = \frac{304r^2 + 304s^2 - 3360r^2s - 1080rs^2 - 20r - 20s + 8100r^2s^2 - 1672rs}{(30rs - 4r - 4s)^2}$

(2) $AL(G) = 8r + 4s - 8$

(3) $IR1(G) = \frac{6360r^2 - 480s^2 - 29940r^2s - 13200rs^2 + 24300r^2s^2 + 5880rs}{(30rs - 4r - 4s)}$

(4) $IR2(G) = \sqrt{\frac{-134r - 132s + 405rs + 4}{45rs - 10r - 10s}} - \frac{90rs - 20r - 20s}{30rs - 4r - 4s}$

(5) $IRF(G) = 4r + 524s - 8.$

***Proof :***

( 1) From Eq. 1.1 and Table 2, we have

$$VAR(G) = \frac{M_1(G)}{k} - \left(\frac{2l}{k}\right)^2$$

$$= \frac{-76r - 76ss + 270rs}{30rs - 4r - 4s} - \left(\frac{2(45rs - 10r - 10s)}{30rs - 4r - 4s}\right)^2$$

$$= \frac{(-76r - 76s + 270rs)(30rs - 4r - 4s) - (90rs - 20r - 20s)}{(30rs - 4r - 4s)^2}$$

$$= \frac{304r^2 + 304s^2 - 3360r^2s - 1080rs^2 - 20r - 20s + 8100r^2s^2 - 1672rs}{(30rs - 4r - 4s)^2}$$

(2) From Eq. 1.2 and Table 2, we get

$$AL(G) = \sum_{gh\in E(G)} |d_g - d_h|$$

$$= |2-2|(4r+6s+4) + |2-3|(8r+4s-8) + |3-3|(45rs - 22r - 20s + 4)$$

$$= 8r + 4s - 8.$$

(3) From Eq. 1.3 and Table 2, we conclude

$$IR1(G) = F(G) - \frac{2l}{k} M_1(G)$$

$$= (-260r - 260s + 810rs)$$

$$- \frac{2(45rs - 10r - 10s)}{(30rs - 4r - 4s)}(-76r - 76s + 270rs)$$

$$= \frac{6360r^2 - 480s^2 - 29940r^2s - 13200rs^2 + 24300r^2s^2 + 5880rs}{(30rs - 4r - 4s)}$$

(4) From Eq. 1.4 and Table 2, we have

$$IR2(G) = \sqrt{\frac{M_2(G)}{l}} - \frac{2l}{k}$$

$$= \sqrt{\frac{-134r - 132s + 450rs + 4}{45rs - 10r - 10s}} - \frac{2(45rs - 10r - 10s)}{30rs - 4r - 4s}$$

$$= \sqrt{\frac{-134r - 132s + 450rs + 4}{45rs - 10r - 10s}} - \frac{90rs - 20r - 20s}{30rs - 4r - 4s}$$

(5) From Eq. 1.5 and Table 2, we get

$$IRF(G) = F(G) - 2M_2(G)$$

$$= -260r - 260s + 810rs - 2(-134r - 132s + 450rs + 4)$$

$$= 4r + 524s - 8.$$

***Theorem***

Suppose *G* represent *L(S(H(r,s)))*, where *L(S(H(r,s)))* be the line graph of subdivision graph of *H*-naphtalenic nano-sheet, then we determine the followings indices:

(1) $IRFW(G) = \frac{4r + 524s - 8}{405rs - 134r - 132s + 4}$

(2) $IRA(G) = \left(\frac{20}{3} - \frac{8\sqrt{2}}{\sqrt{3}}\right)r + \left(\frac{10}{3} - \frac{\sqrt{2}}{\sqrt{3}}\right)s + \left(-\frac{20}{3} + \frac{8\sqrt{2}}{\sqrt{3}}\right)$

(3) $IRB(G) = (40 - 16\sqrt{6})r + (20 - 8\sqrt{6})s + (16\sqrt{6} - 40)$

(4) $IRC(G) = \frac{(-58 + 6\sqrt{8})r + (-48 + 4\sqrt{6})s + 135rs}{45rs - 10r - 10s} - \frac{45rs - 10r - 10s}{15rs - 2r - 2s}$

(5) $IRDIF(G) = \frac{20}{3}r + \frac{10}{3}s - \frac{20}{3}$

***Proof:***

(1) From Eq. 1.6 and Table 2, we have

$$IRFW(G) = \frac{IRF(G)}{M_2(G)}$$

$$= \frac{4r + 524s - 8}{405rs - 134r - 132s + 4}$$

$$= \frac{4r + 524s - 8}{405rs - 134r - 132s + 4}.$$

(2) From Eq. 1.7 and Table 2, we get

$$IRA(G) = \sum_{gh\in E(G)} \left(d_g^{-1/2} - d_h^{-1/2}\right)^2$$

$$= \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^2 (4r + 6s + 4) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)^2 (8r + 4s - 8)$$

$$+ \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)^2 (45rs - 22r - 20s + 4)$$

$$= \left(\frac{20}{3} - \frac{8\sqrt{2}}{\sqrt{3}}\right)r + \left(\frac{10}{3} - \frac{\sqrt{2}}{\sqrt{3}}\right)s + \left(-\frac{20}{3} + \frac{8\sqrt{2}}{\sqrt{3}}\right)$$

(3) From Eq. 1.8 and Table 2, we conclude

$$IRB(G) = \sum_{gh\in E(G)} \left(d_g^{1/2} - d_h^{1/2}\right)^2$$

$$= (\sqrt{2} - \sqrt{2})^2 (4r + 6s + 4) + (\sqrt{2} - \sqrt{3})^2 (8r + 4s - 8) + (\sqrt{3} - \sqrt{3})^2 (45rs - 22r - 20s + 4)$$

$$= (40 - 16\sqrt{6})r + (20 - 8\sqrt{6})s + (16\sqrt{6} - 40).$$

(4) From Eq. 1.9 and Table 2, we have

$$IRB(G) = \sum_{gh\in E(G)} \left(d_g^{1/2} - d_h^{1/2}\right)^2$$

$$= (\sqrt{2} - \sqrt{2})^2 (4r + 6s + 4) + (\sqrt{2} - \sqrt{3})^2 (8r + 4s - 8) + (\sqrt{3} - \sqrt{3})^2 (45rs - 22r - 20s + 4)$$

$$= (40 - 16\sqrt{6})r + (20 - 8\sqrt{6})s + (16\sqrt{6} - 40).$$

(5) From Eq. 1.10 and Table 2, we get

$$IRDIF(G) = \sum_{gh\in E(G)} \left|\frac{d_g}{d_h} - \frac{d_h}{d_g}\right|$$

$$= \left|\frac{2}{2} - \frac{2}{2}\right| (4r + 6s + 4) + \left|\frac{2}{3} - \frac{3}{2}\right| (8r + 4s - 8) + \left|\frac{3}{3} - \frac{3}{3}\right| (45rs - 22r - 20s + 4)$$

$$= \left(\frac{20}{3}r - \frac{10}{3}s - \frac{20}{3}\right).$$

***Theorem***

Suppose *G* represent *L(S(H(r,s)))*, where *L(S(H(r,s)))* be the line graph of subdivision graph of H-naphtalenic nano-sheet, then we determine the followings indices:

(1) $IRL(G) = 3.2436r + 1.6218s - 3.2436$

(2) $IRLU(G) = 4r + 2s - 4$

(3) $IRLF(G) = \frac{1}{\sqrt{6}}(8r + 4s - 8)$

(4) $IRLA(G) = \frac{1}{5}(8r + 4s - 8)$

(5) $IRDI(G) = 5.5448r + 2.7724s - 5.5448$

(6) $IRGA(G) = 0.16328r + 0.08164s - 0.16328.$

***Proof:***

(1) From Eq. 1.11 and Table 2, we have

$$IRL(G) = \sum_{gh\in E(G)} |lnd_g - lnd_h|$$

$$= |ln2 - ln2|(4r + 6s + 4) + |ln2 - ln3|(8r + 4s - 8) + |ln3 - ln3|(45rs - 22r - 20s + 4)$$

$$= (3.2436r + 1.6218s - 3.2436).$$

(2) From Eq. 1.12 and Table 2, we get

$$IRDLU(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{min(d_g, d_g)}$$

$$= \frac{|2-2|}{min(2,2)}(4r + 6s + 4) + \frac{|2-3|}{min(2,3)}(8r + 4s - 8) + \frac{|3-3|}{min(4,4)}(45rs - 22r - 20s + 4)$$

$$= (4r + 2s - 4).$$

(3) From Eq. 1.13 and Table 2, we deduce

$$IRDLF(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{\sqrt{d_g d_h}}$$

$$= \frac{|2-2|}{2+2}(4r + 6s + 4) + \frac{|2-3|}{2+3}(8r + 4s - 8) + \frac{|3-3|}{3+3}(45rs - 22r - 20s + 4)$$

$$= \frac{1}{\sqrt{6}}(8r + 4s - 8).$$

(4) From Eq. 1.14 and Table 2, we have

$$IRLA(G) = \sum_{gh\in E(G)} \frac{|d_g - d_h|}{d_g + d_h}$$

$$= \frac{|2-2|}{2+2}(4r + 6s + 4) + \frac{|2-3|}{2+3}(8r + 4s - 8) + \frac{|3-3|}{3+3}(45rs - 22r - 20s + 4)$$

$$= \frac{1}{6}(8r + 4s - 8).$$

(5) From Eq. 1.15 and Table 2, we get

$$IRDI(G) = \sum_{gh\in E(G)} ln\{1 + |d_g - d_h|\}$$

$$= ln\{1 + |2-2|\}(4r + 6s + 4) + ln\{1 + |2-3|\}(8r + 4s - 8) + ln\{1 + |3-3|\}(45rs - 22r - 20s + 4)$$

$$= 0.16328r + 0.08164s - 0.16328$$

(6) From Eq. 1.16 and Table 2, we conclude

$$IRGA(G) = \sum_{gh\in E(G)} \frac{ln(d_g + d_h)}{2\sqrt{d_g d_h}}$$

$$= \frac{ln(2+2)}{2\sqrt{2\times 2}}(4r + 6s + 4) + \frac{ln(2+3)}{2\sqrt{2\times 3}}(8r + 4s - 8) + \frac{ln(3+3)}{2\sqrt{3\times 3}}(45rs - 22r - 20s + 4)$$

$$= 0.16328r + 0.08164s - 0.16328.$$

We have sketched the irregular indices graphically for the indium Phosphide *InP[m,n]* and line graph of subdivision graph of *H*-naphtalenicnano-sheet *L(S(H(r,s)))* and quantitative comparison is shown in Fig. 3 (a) to (j). It indicates how the irregular indices change its behavior with the increase in network size and structure. Graphical representation of irregular indices for some *InP[m,n]* and *L(S(H(m,n)))*is shown below.

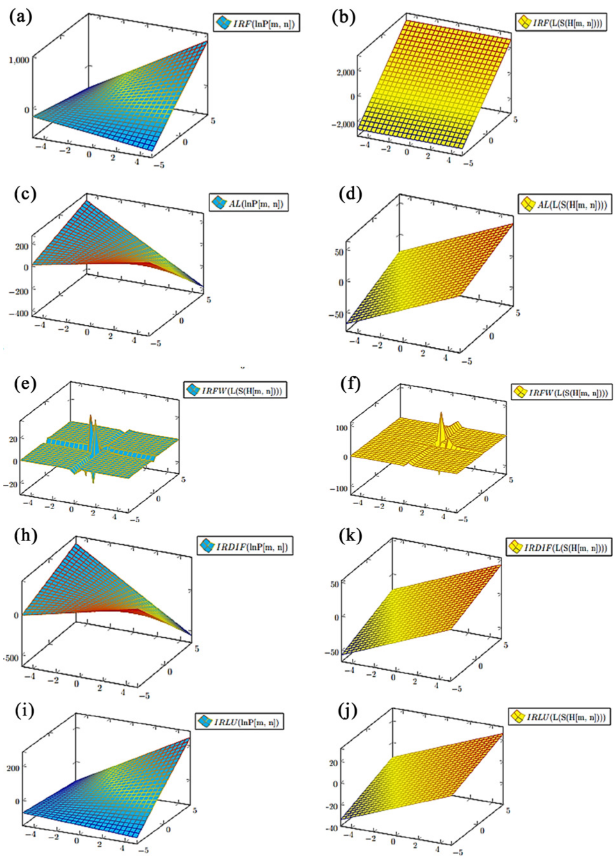


Fig. 3. (a) to (j) Irregular indices for Indium Phosphide *(InP[m,n]* and the line graph of the subdivision graph of H-naphtalenic nanosheet *L(SH(r,s)))*

## CONCLUSIONS

We have determined the irregular indices of indium Phosphide *InP[m,n]* and line graph of subdivision graph of *H*-Naphtalenic nano-sheet L(S(H(r,s))) and quantitative comparison is shown in figures. The results show which properties have most effective on invariants. Molecular characteristics like: flash point, boiling point, critical pressure and polarization etc. can be computed by using topological indices. We have also investigated different indices for indium Phosphide *InP[m,n]* and line graph of subdivision graph of *H*-Naphtalenic nano-sheet L(S(H(r,s))) such as VAR(G), AL(G), IR1(G), IR2(G), IRF(G), IRFW(G), IRA(G), IRB(G), IRC(G), IRDIF(G), IRL(G), IRLU(G), IRLF(G), IRLA(G), IRDI(G), IRGA(G). Graphical behaviors of InP[m,n] and L(S(H(r,s)))have also been discussed at the end.

## AUTHOR CONTRIBUTIONS:

All authors contributed equally to this study.

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All data used are included inside the article.

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