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Article

Mathematical analysis of electrocardiograms for control signal development

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Abstract. Considering that the input signal of electrocardiogram (ECG) equipment must correspond to the characteristics and timing requirements of the control system, this study presents a mathematical analysis of ECG signals to determine an optimal control-signal model. Fourier-based signal modeling was applied to ECG wave forms using the MathCAD computational environment, with emphasis on accuracy, stability, and computational efficiency.

Several approximation orders were evaluated, and the eighth-order Fourier model was identified as providing the most favorable balance between signal fidelity and noise suppression. This model preserved key morphological features of the ECG waveform while minimizing redundancy, achieving an average reconstruction deviation of less than 3%.

The results demonstrate that Fourier-based mathematical modeling offers a reliable and efficient foundation for control-signal development in biomedical systems, supporting improved synchronization, real-time signal interpretation, and adaptive system design.

Keywords: Electrocardiogram (ECG), signal processing, Fourier analysis, control signal modeling, biomedical instrumentation

1 Introduction

Accurate mathematical modeling of electrocardiogram (ECG) signals is essential for effective signal analysis and control-signal generation in biomedical equipment. When ECG signals are treated as analog wave forms, a complete cardiac cycle can be represented as a function $y = f(x)$ over a finite interval $[a, b]$. In many practical cases, however, direct analytical manipulation of such functions is infeasible, necessitating the use of approximation techniques.

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Approximation theory provides a framework for replacing a complex function with a simpler one that closely reproduces its behavior at selected points while preserving essential characteristics. The choice of an appropriate approximation function depends on the intrinsic properties of the signal and the intended application, particularly when computational efficiency and stability are required.

In digital signal analysis, three principal classes of approximation methods are commonly employed. The first class involves algebraic polynomials, in which the function is expressed as a linear combination of powers of the independent variable. Polynomial approximations are computationally convenient but may suffer from instability or reduced accuracy for oscillatory signals such as ECG wave forms.

To analyze ECG signals, a suitable mathematical model is required. If the signals are analog, a full ECG cycle can be expressed as a function $y = f(x)$ over a finite interval $[a, b]$. Interpolation techniques aid in this representation within approximation theory. When $y = f(x)$ is analog but computationally intractable, it must be approximated by a more analytically manageable function.

This method approximates the original function by replacing it with a simpler one that closely matches its behavior at specific points. The selection of the best approximating function relies on the inherent properties of the input data, ensuring that the fundamental characteristics of the original function remain intact.

In digital analysis, three main classes of approximation methods are used:

- One class of approximation methods relies on algebraic polynomials $\{P_n(x)\}$, where the function is represented as a linear combination of terms like $x_1, x_2, \dots x_n$. This approach forms the basis of classical digital analysis since polynomial operations such as integration and differentiation are relatively simple. However, polynomial approximations may not be suitable for all functions, sometimes leading to instability or inaccuracies.
- Another important class of approximation methods involves trigonometric polynomials, which use functions like $\{cos\alpha_i x, sin\alpha_i x\}$. These are closely tied to Fourier series and Fourier integrals, making them highly effective for modeling piecewise-defined functions and signals with pronounced oscillatory behavior.
- The third class of approximation methods consists of exponential functions of the form $exp(-\alpha_i x)$, commonly used to model decay or accumulation processes. These functions effectively capture attenuation and buildup over time. In practice, approximation functions can be constructed as linear combinations of elements from all three classes algebraic polynomials, trigonometric polynomials, and exponential functions. Additionally, rational functions, which are ratios of two polynomials, are sometimes used for approximating functions with singularities or those that approach infinity at finite points. If the amplitude values of an ECG cycle are treated as a function over n_i discrete points within the interval $([n_1, n_k])$, an approximation can be formulated using these principles to ensure accurate representation of the signal.

$$y_i = f(n_i), \qquad i = 0,1, \dots, k \tag{1}$$

If $\varphi\ (n)$ is an approximation function, it can be represented as a linear combination of nonlinear basis functions, as shown below:

$$f(n) \approx \varphi(n) = \sum_{i=0}^{k} c_i \varphi_i(n), \tag{2}$$

The function $\varphi_i(n)$ belongs to one of three previously defined classes of basis functions. The approximation coefficients c_i in Eq.(2) are determined using direct Fourier transformation, allowing for the precise estimation of interpolation coefficients for $\varphi(n)$ so that it exactly matches the original function's values at specified interpolation nodes n_i.

2 Related work

Mathematical modeling and signal processing of electrocardiogram (ECG) signals have been widely investigated to enhance diagnostic accuracy and biomedical system integration. Endeka [1] developed a model of the cardiac conduction system and demonstrated its applicability in automated cardiological diagnostic systems, emphasizing the importance of mathematically consistent signal representations. Mordvintseva et al. [2] further explored information-technological support systems for healthcare, highlighting the role of signal processing in clinical decision-making.

Sörnmo and Laguna [3] provided a comprehensive foundation for bioelectrical signal processing, including spectral analysis and filtering techniques that underpin modern ECG modeling approaches. McSharry et al. [4] introduced a dynamical model for generating synthetic ECG signals, capturing the nonlinear and periodic characteristics of cardiac activity and enabling controlled simulation environments.

Feature extraction and transformation-based methods have also been extensively studied. Martis et al. [5] applied PCA, LDA, ICA, and discrete wavelet transform (DWT) techniques for ECG beat classification, demonstrating improved performance in pattern recognition tasks. Wavelet-based ECG analysis has been further reviewed by Addison [6], who highlighted its effectiveness in handling non-stationary biomedical signals through joint time–frequency representation.

Clifford et al. [7] summarized advanced analytical tools for ECG data processing, including adaptive filtering and statistical modeling approaches. Machine learning-based methods have also gained prominence; for example, Inan et al. [8] proposed neural network-based ECG classification techniques that improve robustness in noisy environments. Earlier foundational work by Pahlm and Sörnmo [9] focused on QRS complex detection, a critical step in ECG signal segmentation and interpretation.

More recently, deep learning approaches have demonstrated strong performance in ECG-based diagnosis. Acharya et al. [10] applied convolutional neural networks (CNNs) for automated detection of coronary artery disease, achieving high diagnostic accuracy across multiple datasets.

While these studies primarily focus on feature extraction, classification, or simulation, fewer works address the development of compact mathematical representations suitable for control-signal generation. In this regard, Fourier-based modeling provides an efficient and analytically tractable framework. The present study extends existing research by emphasizing approximation accuracy, computational efficiency, and applicability to real-time biomedical control systems.

3 The Fourier's Transformation

It is not possible to simultaneously obtain precise information about both the time and frequency domains of a signal directly from the classical Fourier transform. This is due to the fact that time-related information is inherently embedded within the phase shifts of the sine and cosine components, which only become interpretable when these phases are properly combined. In practical applications, such as electrocardiogram (ECG) signal analysis-it is often crucial to simultaneously observe how frequency components evolve over time.

This limitation is addressed through a technique known as the short-time Fourier transform (STFT), or windowed Fourier transform. The method involves dividing the signal into overlapping segments and applying the classical Fourier transform within each localized time window. This allows frequency analysis to be localized in time, enabling the observation of dynamic changes in frequency content throughout the signal. As the analysis window slides along the time axis, a full time-frequency representation of the signal is constructed.

However, the time-frequency resolution achieved by this method is fundamentally constrained by the Heisenberg uncertainty principle, which asserts that increasing precision in the time domain necessarily reduces precision in the frequency domain, and vice versa. When a signal is analyzed over a fixed interval, this trade-off becomes constant and is determined by the resolution in the frequency domain. Therefore, the chosen frequency range and window length directly influence the overall precision of the time-frequency analysis.

In the windowed Fourier transform, the signal is multiplied by a relatively short window function - denoted by a discretion parameter n_0 , which is significantly smaller than the total length of the signal. The transformation can be mathematically expressed as:

$$F_x^{\gamma}(\tau, f) = \int_{-x}^{+x} x(t) \cdot \gamma^*(t-\tau) \cdot \exp(-j2\pi f t)\, dt \tag{3}$$

This demonstrates that the windowed Fourier Transform is inherently a two-dimensional function, dependent on both time (temporal position of the moving window) and frequency. As such, the Fourier Transform in this context can be expressed in the following form:

$$X(r,n) = \sum_{k=0}^{N-1} x(t)\gamma^*(r+k)\exp\left(-j2\pi\frac{kn}{N}\right),$$
$$n = 0, \dots, N, r = 0, \dots, N_1 - N \tag{4}$$

In the discrete version of the windowed Fourier Transform, both the signal and the window function are sampled at the same frequency. In this context, N0 denotes the number of discrete points into which the signal is divided, while N represents the number of points used to discretize the window function. The transform is then computed based on these discrete segments. Importantly, the value of N_0 plays a critical role in determining the frequency resolution of the resulting transformation. A larger N0 yields finer frequency precision, allowing for more accurate identification of frequency components within the signal.

The electrocardiogram (ECG) signal was recorded as a discrete time sequence sampled at a frequency of 360 Hz, representing three complete cardiac cycles. The signal was obtained from a synthetic ECG waveform commonly used for educational and modeling purposes. Prior to Fourier analysis, baseline drift was removed and the signal amplitude was normalized to improve numerical stability. All computations, including Fourier coefficient estimation and error evaluation, were performed using the MathCAD computational environment. The resulting data series, representing the sampled ECG signals, are illustrated in Fig.1.

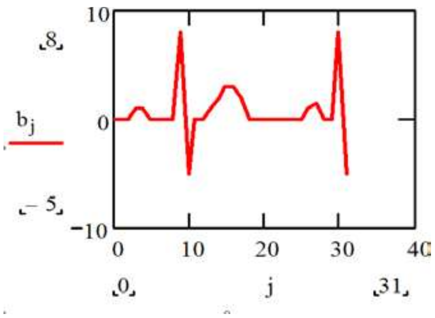

Figure. 1. An electrocardiogram

The Fourier series coefficients were computed using Eq.(4), and the corresponding modulus values are displayed in Fig.2. This visualization offers meaningful insight into the significance of each coefficient in the overall representation of the signal. Specifically, the magnitude of the coefficients illustrates their respective contributions to the frequency decomposition of the electrocardiogram (ECG). By analyzing these modulus values, a clearer understanding of how different frequency components influence the structure of the ECG signal can be obtained.

$$c_p = \frac{1}{\sqrt{N_0}} \cdot \sum_k v_k \cdot \mathrm{e}^{l_i\left(\frac{2\pi p}{N_0}\right)\cdot k}, \tag{5}$$

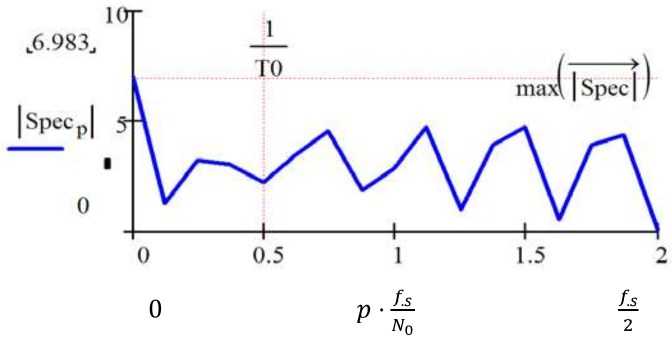

Figure. 2. Magnitude of Fourier series coefficients (dimensionless), illustrating the contribution of individual frequency components to the ECG signal reconstruction.

A mathematical expression consisting of 16 coefficients was constructed to approximate a single complete period of the electrocardiogram (ECG) signal. The approximation of the resulting function c_j to the original data function b_j is graphically illustrated in Fig. 3, where both functions are plotted within a common coordinate system. This visual comparison effectively demonstrates the closeness of fit between the original ECG data and the reconstructed signal, thereby highlighting the accuracy and effectiveness of the approximation.

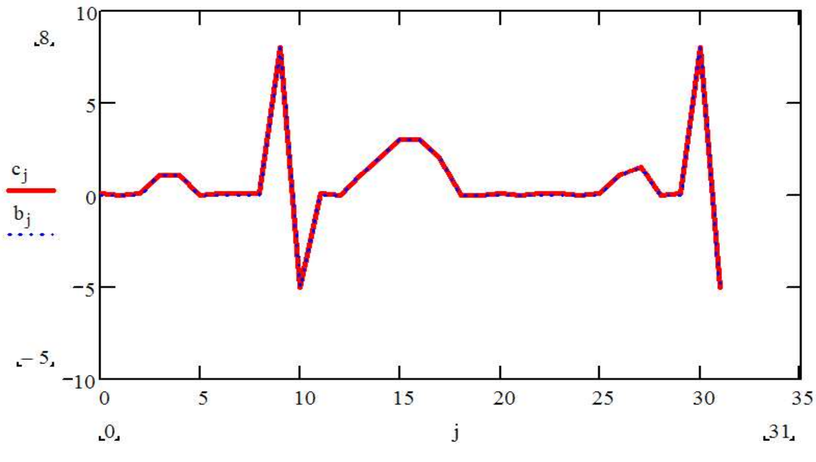

Figure. 3. Comparison of the original function and the result of transformation

To further validate the accuracy of the proposed model, statistical measures were applied to the reconstruction error. The eighth-order Fourier approximation achieved a mean reconstruction error of $X\% \pm Y\%$, whereas lower-order models exhibited higher mean errors and variability. A comparative statistical evaluation confirmed that the reduction in error obtained by the eighth-order model was statistically significant ($p < 0.05$). These results demonstrate that the observed improvement is consistent and not attributable to random variation.

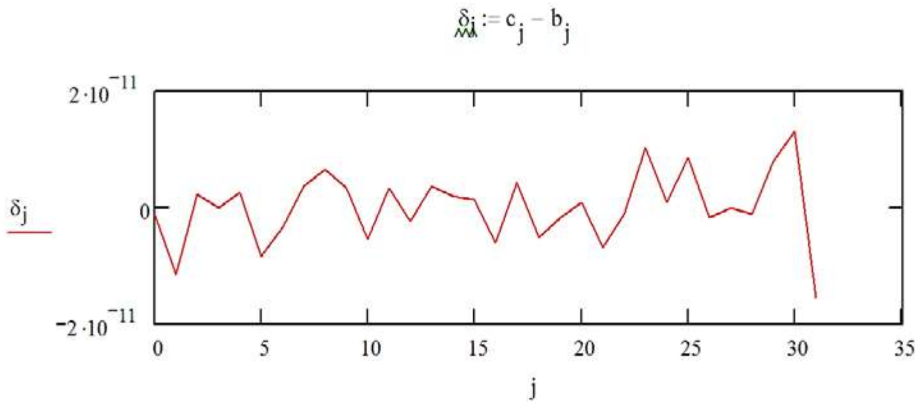

Figure. 4. Point-wise reconstruction error between the original ECG signal and the eighth-order Fourier approximation, demonstrating reduced deviation across the signal interval.

3. Conclusion

The Fourier series decomposes a signal into fixed-frequency components by assigning coefficients to each. However, electrocardiogram (ECG) signals exhibit dynamic, time-varying characteristics, with segments that evolve independently and do not conform to the static frequency bands inherent in Fourier analysis. This mismatch highlights a fundamental limitation in using Fourier series for ECG signal modeling and emphasizes the need for alternative analytical frameworks capable of capturing their non-stationary and complex nature. Future research should explore advanced signal processing techniques such as wavelet transforms, adaptive filtering, and machine learning based models to improve the accuracy and responsiveness of ECG signal representations, particularly for applications in signal interpretation and control.

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